Double-logarithmic behavior of inelastic fermion form factors in QED and QCD

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The effective kinematic diagram technique is applied to study inelastic form factors of electron and quark in QED and QCD. The explicit expressions for these form factors in the double-logarithmic approximation are presented. The self-consistency of the results is shown by demonstrating the fulfillment of the Kinoshita-Lee-Nauenberg theorem.

I. INTRODUCTION

Precise computation of the elastic and inelastic fermion form factors in hard collisions is required to test the predictive power of the Standard Model, as well as the effective and unambiguous detection of signals of New Physics at modern and future colliders (see, *e.g.*, [1] and references therein). In QCD, the quark form factors are used in calculations of various QCD processes at the partonic level, and are of a considerable phenomenological importance [2]. In investigation of e^+e^- collisions at TeV energies, the resumed leading and nonleading Sudakov corrections to fermion form factors may profoundly influence the cross sections, and play a significant role in calculations for the Next Linear Collider [6, 7, 8].

Since the pioneering calculation of the resumed double-logarithmic (DL) corrections to the elastic electron form factor in QED [3], significant progress has been made in evaluation of the next-to-leading logarithmic contributions [4, 5], as well as in generalization of these results to the strong (QCD) and electroweak (EW) sectors of the Standard Model (see, *e.g.*, [6, 7, 9], and references therein).

The well-known Sudakov elastic form factor $F(q^2)$ of the electron scattering in external electromagnetic field with large transferred momentum $q = p_2 - p_1$

$$\mathrm{e}(p_1) + \gamma^*(q) \to \mathrm{e}(p_2) \tag{1}$$

has the form [3]

$$F(s) = \exp\left(-\frac{\alpha}{4\pi}\ \ln^2\frac{s}{\lambda^2}\right)\ ,\ s \gg \lambda^2\ , \tag{2}$$

where $s = -q^2$ and the mass λ of the virtual vector boson is introduced in order to regulate the IR divergence.

Such a strong suppression of elastic form factor is quite natural since it reflects a small probability for an electron to remain to be itself in this process. Therefore, inelastic processes with emission of one or several real vector bosons become more probable. Although all the exclusive scattering probabilities experience the Sudakov type suppression, the total sum of them must be equal to 1 and possess no singularities in the massless limit in accordance with the Kinoshita-Lee-Nauenberg (KLN) theorem. Such a cancellation can be easily proven in QED using the Poisson nature of the inelastic form factor. As for QCD, the problem of KLN cancellation is more complicated due to violation of the Poisson form of form factors. Nevertheless, the explicit expressions for inelastic form factors with radiative corrections taken into account can be obtained in certain kinematics of the real gluon emission which can be realized in experiment. An effective way to get them is to apply the kinematic diagram method. It is the motivation of our paper to calculate the inelastic fermion form factors within this framework.

II. DESCRIPTION OF THE METHOD

In this paper, we derive the inelastic form factor of a fermion (electron, or quark) in the double-logarithmic (DL) approximation

$$g^2 \ll 1\ ,\quad g^2L^2 \gg 1\ ,\quad L = \ln\frac{s}{\lambda^2},\quad s = -q^2 \gg \lambda^2\ , \tag{3}$$

where λ is the mass of a virtual vector boson, and $g^2 = 4\pi\alpha$. A powerful method for calculation of the elastic cross sections in this approximation was developed in the Quantum ElectroDynamics (QED) and in the Quantum ChromoDynamics (QCD) (see [10] and references therein). It was found out that the straightforward calculation of the Feynman diagrams was not the most economical way to resume the DL asymptotics of form factors and cross sections. Here we apply a different approach based on the use of the effective kinematic diagrams.

Throughout the paper we use the Sudakov representation of the four-momenta of the virtual bosons (neutral massive vector particles, or massive gluons)

$$k = \alpha p_2 + \beta p_1 + k_\perp\ , \tag{4}$$

where $p_1,\ p_2$ are the four-momenta of the external fermions, and $q = p_2 - p_1$ is the transferred momentum in the scattering channel. Let us remind the main features

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of the Sudakov parameterization

$$k_\perp p_1 = k_\perp p_2 = 0\ , \quad k^2 = s\alpha\beta - \boldsymbol{k}^2\ , \qquad (5)$$
$$k_{i\perp}k_{j\perp} = -\boldsymbol{k}_i\boldsymbol{k}_j\ , \quad \mathrm{d}^4k = \frac{s}{2}\,\mathrm{d}\alpha\,\mathrm{d}\beta\,\mathrm{d}^2\boldsymbol{k}\ .$$

The ground of the effective kinematic diagram method is twofold. The first reason is the strong ordering of the virtual photons in magnitude of transversal components of their four-momenta. For a set of Feynman diagrams (FD) with n virtual vector bosons, the main DL contribution arises from the region where their transversal momenta are strictly ordered

$$s \gg \boldsymbol{k}_{i_1}^2 \gg \boldsymbol{k}_{i_2}^2 \gg \cdots \gg \boldsymbol{k}_{i_n}^2 \gg \lambda^2\ . \qquad (6)$$

The second reason is the Gribov theorem about validity of the classical current approximation for the emission of vector bosons in the extended region [11]. The main idea of this theorem (in a particular case of the DL calculations) can be expressed in the following manner: The amplitude of the process $q(p) + g(k) \to X$ can be related with the amplitude of the transition of the almost on-mass-shell quark $q(p)$ to the same state X

$$\mathcal{M}\left(q(p) + g(k) \to X\right) = \frac{\epsilon \cdot p}{k \cdot p}\mathcal{M}\left(q(p^*) \to X\right)\ , \qquad (7)$$

if the transversal component of its 4-momentum is small in comparison with the characteristic transversal momentum in the block X, where ϵ is the polarization vector of the gluon and $(p^*)^2 \approx m^2$. Thus, the Gribov theorem refutes the common belief that the classical current approximation is valid only for soft photons.

Consider now the interaction of a virtual gluon having minimal transversal momentum $\boldsymbol{k_m}$ with a quark of momentum p_1 (Fig. (1a)). The corresponding amplitude being an analytical function of the variable $(p + k_m)^2$ has the pole corresponding to the one-quark intermediate state and the cut which corresponds to the quark-gluon intermediate state (Fig. (1b)). Let us now prove that the contribution of the cut is suppressed in the DL approximation while it can contribute to the nonleading terms which do not survive in the asymptotic regime. Indeed, keeping in the mind the current conservation condition for the amplitude of the block $g + q \to X$ [10, 11]

$$k^\mu \mathcal{M}_\mu^X = (\alpha p_2 + k_\perp)^\mu\, \mathcal{M}_\mu^X = 0\ ,$$

and the Green function of a vector boson with momentum k:

$$G_{\mu\nu} = \frac{g_{\mu\nu}^\perp + \frac{2}{s}\left(p_{1\mu}p_{2\nu} + p_{2\mu}p_{1\nu}\right)}{(k^2 - \lambda^2 + i0)}\ , \qquad (8)$$

(there are no ghosts in this gauge) one finds that the cut contribution is associated with the additional factor depending on $|\boldsymbol{k}|/|\boldsymbol{k}_i| \ll 1$ compared with the contribution of the pole FD.

Therefore, in the DL approximation the softest virtual photon (gluon) effectively interacts with the quarks having the momenta p_1 and p_2, *i.e.*, it is emitted before all the photons (gluons) counting along the quark line, and absorbed after all other gluons. Similar reasons lead to construction of a ladder type FD with all the rungs parallel to each other. In calculation of the corresponding amplitude it is implied that the virtualities of the vector bosons are strictly ordered (Fig. (1c)).

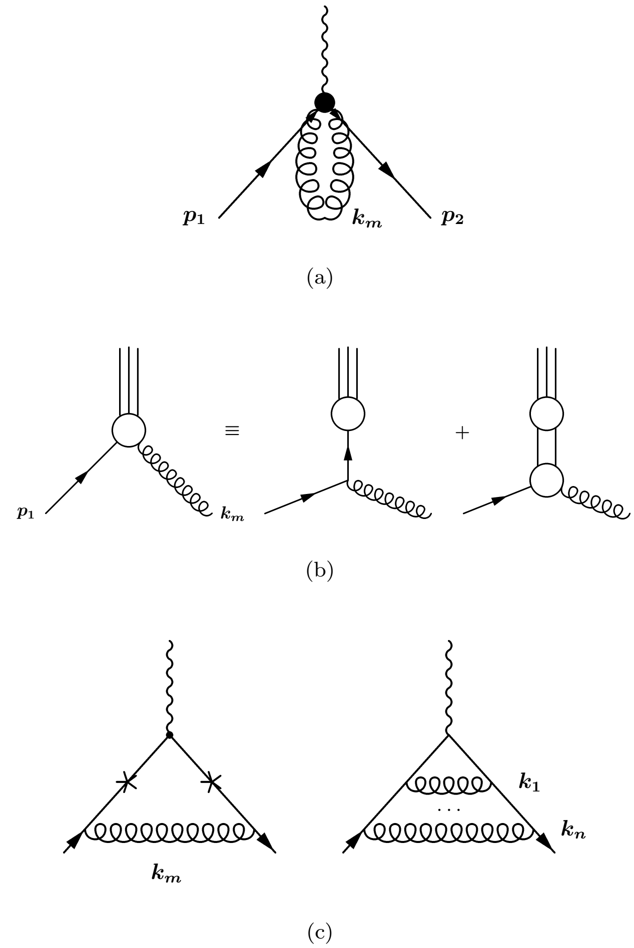

Fig. 1: The kinematic diagrams for (a) the extracted gluon g_{k_m} with the minimal value of the transversal momentum $\boldsymbol{k}_m$, (b) the pole and cut contributions to the quark-gluon amplitude, (c) the pole dominant kinematic diagrams with descending momenta $\boldsymbol{k}_1^2 \gg \boldsymbol{k}_2^2 \gg \cdots \gg \boldsymbol{k}_n^2$.

It is easy to understand that due to this ordering the contributions of such FD can be expressed in terms of the lowest order (Born) amplitude $B^{(0)}$ as ${B^{(0)}}^n/n!$, which leads to the Sudakov type form factor

$$F^{(0)}(s) = \mathrm{e}^{-B^{(0)}}\ . \qquad (9)$$

The condition of ordering can be removed when one considers the whole set of $n!$ similar expressions obtained by symmetrization of the momenta indices. Thus, the combinatorial factor $1/n!$ must be introduced.

This type of the DL behavior can be obtained by explicit calculations in lowest orders of PT in both QED

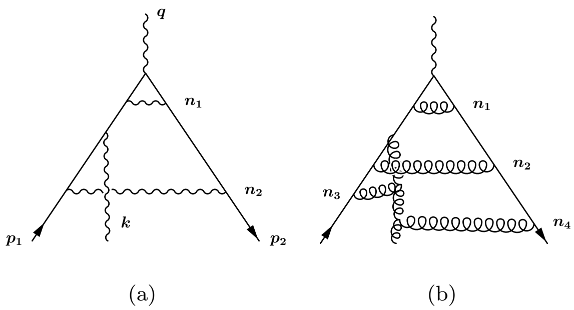

Fig. 2: The QED kinematic diagram for the hard photon emission (a) and the QCD kinematic diagram for the hard gluon emission (b).

[13], $B^{(0)}_{\rm QED} = \left(e^2/16\pi^2\right)\ L^2$, and QCD [14], $B^{(0)}_{\rm QCD} = (\alpha_s/4\pi)\, C_F L^2$. In Appendix we derive these lowest order expressions.

It can be shown that a possible contribution of longitudinal polarized virtual and real vector particles is suppressed due to the gauge invariance and is irrelevant in the DL regime.

III. INELASTIC FORM FACTORS FOR ONE VECTOR BOSON EMISSION

Now let us consider the inelastic electron form factor which includes the emission of a photon with momentum k_1 and polarization vector $\epsilon(k_1)$ (Fig. (2a)). We consider the situation when the transversal momentum of this real hard photon is large compared to the virtual photon mass λ and the masses of fermions:

$$\boldsymbol{k}_1^2 = s\alpha_1\beta_1 \gg \lambda^2, m^2\ . \tag{10}$$

This corresponds to the kinematics which produces the main contribution to the total cross section. Indeed, consider for estimation the contribution to the total cross section in the classical current approximation:

$$-\int \frac{\mathrm{d}^3 k_1}{\omega_1} j^2(k_1) \sim \int\limits_0^{\pm 1}\int\limits_0^{\pm 1} \frac{\mathrm{d}\alpha_1}{\alpha_1}\frac{\mathrm{d}\beta_1}{\beta_1}\theta(s\alpha_1\beta_1-\lambda^2) = L^2\ , \tag{11}$$

$$j^\mu(k_1) = \left(\frac{p_1}{p_1 k_1} - \frac{p_2}{p_2 k_1}\right)^\mu\ .$$

Again, the effective kinematic ladder FD approach can be applied, but the large magnitude of $\boldsymbol{k}_1^2$ requires some modification in the ordering procedure. Namely, for the virtual photons emitted above the point where the real photon is emitted (which is closer to the point of interaction with the external particle) we must choose the quantity $\boldsymbol{k}_1^2$ as a lower bound for $\boldsymbol{k}_i^2$. The virtual photons emitted below the point of the external photon emission have $\boldsymbol{k}_1^2$ as an upper bound. Therefore, the restriction has the following form $\lambda^2 \ll \boldsymbol{k}_j^2 \ll \boldsymbol{k}_1^2$. Denoting the number of the "up" photons by n_1 and the "down" ones by $n-n_1$, one obtains the contribution of n virtual photons to the amplitude of the one-photon radiative scattering

$$\mathcal{M}^{(1)}_n = eV_0 j^\mu(k_1)\epsilon_\mu(k_1)\left(-\frac{e^2}{16\pi^2}\right)^n \sum_{n_1=0}^{n} \frac{(L_1^2)^{n_1}}{(n_1)!}\frac{\left(L^2-L_1^2\right)^{n-n_1}}{(n-n_1)!} = eV_0 j^\mu(k_1)\epsilon_\mu(k_1)\frac{\left(-\frac{e^2L^2}{16\pi^2}\right)^n}{n!}\ , \tag{12}$$

where

$$L_1 = \ln\frac{s}{\boldsymbol{k}_1^2}\ , \quad V_0 = e\bar{u}(p_2)\Gamma u(p_1)\ , \quad \Gamma = (1;\gamma_5;\gamma_\rho;\gamma_5\gamma_\rho)\ .$$

Further resummation is straightforward.

In the case of emission of k real hard photons we have

$$\mathcal{M}^{(k)}_\infty = e^k\ V_0 \prod_{j=1}^{k} j^\mu(k_j)\epsilon_\mu(k_j)\mathrm{e}^{-B^{(0)}/2}\ . \tag{13}$$

It is implied that a hard photon is a photon with transversal momentum much larger than masses of fermions and virtual photons. The contribution to the total cross section is associated with the factor

$$F^{(k)}_\infty = \frac{{B^{(0)}}^k}{k!}\mathrm{e}^{-B^{(0)}}\ , \tag{14}$$

confirming the Poisson nature of the neutral vector bosons emission. The factor $1/k!$ takes into account the identity of the emitted bosons. Therefore, we can see that the Poisson distribution is valid not only for the soft photons [13], but also for the hard ones in the DL approximation [15].

In order to establish the consistency of the result we verify the fulfillment of the KLN theorem [16] about cancellation of the mass singularities, namely

$$\sum_{n=0}^{\infty} F^{(n)}_\infty = 1\ . \tag{15}$$

In QCD, the ladder approach for calculation of the inelastic form factors with emission of a single gluon works as well. However, now the virtual ladder gluons can interact (besides the quarks) with one real hard gluon with momentum k_1. Let us show that in this case the inelastic form factor has the form

$$F_\infty^{(1)} = F_0^{(1)} \exp\left[-\frac{\alpha_s}{4\pi}\left(C_F\ L^2 + \frac{C_V}{2} L_k^2\right)\right] , \qquad (16)$$
$$F_0^{(1)} = g\ V_{0b}\ j^\mu(k_1)\epsilon_\mu(k_1) , \quad L_k = \ln\frac{\boldsymbol{k}_1^2}{\lambda^2} .$$

where $V_{0b} = \bar{u}(p_2)\sigma_b\Gamma u(p_1)$ is the corresponding Born amplitude, $C_F = \frac{N_c^2-1}{2N_c}$, $C_V = N_c$ are the Casimir operators of the color group $SU(N_c)$, and σ_b's are the group generators. The eight kinematic FD exist in the one-loop order. It is sufficient to consider only four of them which describe the emission from the quark 1 (for simplicity, we denote the quark with the momentum $p_{1,2}$ by "quark 1,2"). The color factor associated with the region when the loop momentum $|\boldsymbol{k}|$ is large as compared with $|\boldsymbol{k}_1|$ is $\sigma_a\sigma_a = C_F\ I$, since the color generators commute with the external vertex operator Γ. This gives the contribution similar to the QED case:

$$-Z_1 C_F L_k^2, \quad Z_1 = gV_{0b}\frac{p_1\cdot\epsilon_1}{p_1\cdot k_1}\frac{\alpha_s}{4\pi} , \quad \epsilon_1 = \epsilon(k_1) . \quad (17)$$

The contribution of another QED-type FD corresponding to the case $|\boldsymbol{k}_1| \gg |\boldsymbol{k}|$ is accompanied by the color factor $\sigma_a\sigma_b\sigma_a = (C_F - C_V/2)\sigma_b$. Its contribution reads

$$-Z_1\left(C_F - \frac{C_V}{2}\right)\left(L^2 - L_k^2\right) . \qquad (18)$$

The color factor of the FD that corresponds to the case $|\boldsymbol{k}_1| \gg |\boldsymbol{k}|$, when the gluon is emitted by the quark 1 and absorbed by the external gluon, is $if_{abc}\sigma_a\sigma_c = C_V/2$. The corresponding kinematic contribution without the restriction imposed by the real gluon emission is given by

$$\int\limits_{\frac{\lambda^2}{s}}^{\alpha_1}\frac{\mathrm{d}\alpha}{\alpha}\int\limits_{\frac{\lambda^2}{s\alpha}}^{1}\frac{\mathrm{d}\beta}{\beta} .$$

The restriction consists in the subtraction of a similar expression with replacement $\lambda^2 \to \boldsymbol{k}_1^2$. The total contribution of this FD reads

$$-Z_1\frac{1}{2}C_V\left[\ln^2\frac{s\alpha_1}{\lambda^2} - \ln^2\frac{s\alpha_1}{\boldsymbol{k}_1^2}\right] . \qquad (19)$$

The contribution of the FD with the gluon rung connecting the real gluon with the quark 2 can be obtained from the last expression by means of the replacement $\alpha_1 \to \beta_1$.

The total one-loop contribution to the one-gluon radiative scattering of a quark (including the FD with the real gluon emitted by the quark 2) has the form

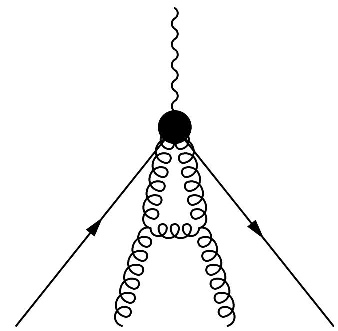

Fig. 3: The kinematic diagram yielding the color exotic contribution.

$$\mathcal{M}_1^{(1)} = gV_{0b}j^\mu(k_1)\epsilon_\mu(k_1)$$
$$\cdot\left[-\frac{\alpha_s}{4\pi}\left(C_F\ L^2 + \frac{1}{2}C_V\ \left(L - L_1\right)^2\right)\right] . \qquad (20)$$

This result agrees with that one obtained in the work by one of us (see Eq. (11) in the Ref. [12]).

The FD with n loops can be parameterized by the numbers n_1, n_2, n_3, n_4 (Fig. (2b)) which are, respectively, n_1—the number of rungs with $\boldsymbol{k}_i^2 \gg \boldsymbol{k}_1^2$; n_2—the number of rungs with $\boldsymbol{k}_1^2 \gg \boldsymbol{k}_i^2$ connecting the quarks, and n_3—the quark 1 with the real gluon ; (n_4)—the number of rungs connecting the real gluon with the quark 2, and $n = n_1 + n_2 + n_3 + n_4$. For the real gluon emitted by the quark 1 one has

$$(-1)^n\sum\frac{1}{n_1!}\left(C_F\ L_k^2\right)^{n_1}\left[\left(C_F - \frac{1}{2}C_V\right)\left(L^2 - L_k^2\right)\right]^{n_2}$$
$$\cdot\frac{1}{n_3!}\left(\frac{1}{2}C_V\ln\frac{\boldsymbol{k}_1^2}{\lambda^2}\ln\frac{s\alpha_1}{\beta_1\lambda^2}\right)^{n_3}$$
$$\cdot\frac{1}{n_4!}\left(\frac{1}{2}C_V\ln\frac{\boldsymbol{k}_1^2}{\lambda^2}\ln\frac{s\beta_1}{\alpha_1\lambda^2}\right)^{n_4}$$
$$= (-1)^n\frac{1}{n!}\left[C_F\ L^2 + \frac{1}{2}C_V\ \left(L - L_1\right)^2\right]^n , \qquad (21)$$

confirming the relation given above (see Eq. (16)).

IV. INELASTIC FORM FACTORS FOR ARBITRARY NUMBER OF EMITTED VECTOR BOSONS

Consider now the emission of two hard gluons. Longitudinal Sudakov parameters of the gluon momenta $k_i = \alpha_i p_2 + \beta_i p_1 + k_{i\perp}$ in the region of main contribution to the cross section obey the following restrictions

$$\frac{\lambda^2}{s} \sim \alpha_1 \ll \alpha_2 \sim 1 , \quad \frac{\lambda^2}{s} \sim \beta_2 \ll \beta_1 \sim 1 . \qquad (22)$$

The corresponding Born amplitude has the form

$$g^2\left[V_{0ab}(\alpha_2\beta_1\gg\alpha_1\beta_2)+V_{0ba}(\alpha_1\beta_2\gg\alpha_2\beta_1)\right] \cdot j^\mu(k_1)\epsilon_\mu(k_1)j^\nu(k_2)\epsilon_\nu(k_2)\ , \quad (23)$$

with $V_{0ab}=\bar{u}(p_2)\Gamma\sigma_a\sigma_b u(p_1)$. Note, that we do not consider here the region $\alpha_2\beta_1=\alpha_1\beta_2$ which also yields the DL contributions to the amplitude as it was shown by rather complicated calculations given in [12]. This kinematic region is specific of QCD and corresponds to the decay of a gluon to two gluons. In [17, 18], the arguments in favour of exponentiation of the DL contributions, including also the decay mechanism, were given.

Consider the emission of both gluons from the quark 1 leg (the similar situation takes place for any other kinematic regions of the two-gluon emission). We have (here and below we use the notation $\sigma_{ij}=\sigma_i\sigma_j$)

$$\frac{p_1\cdot\epsilon_1}{p_1\cdot k_1}\frac{p_1\cdot\epsilon_2}{p_1\cdot k_2}\left[\sigma_{21}\Big|_{\alpha_1\beta_1\gg\alpha_2\beta_2}^{\alpha_2\beta_1\gg\alpha_1\beta_2}+\sigma_{12}\Big|_{\alpha_2\beta_2\gg\alpha_1\beta_1}^{\alpha_1\beta_2\gg\alpha_2\beta_1}\right]\ . \quad (24)$$

For the 1-loop radiative corrections (RC) to this process one needs to distinguish 3 kinematic regions

$$\boldsymbol{q}^2\gg\boldsymbol{k}^2\gg\boldsymbol{k}_2^2\ ,\quad \boldsymbol{k}_2^2\gg\boldsymbol{k}^2\gg\boldsymbol{k}_1^2\ ,\quad \boldsymbol{k}_1^2\gg\boldsymbol{k}^2\gg\lambda^2\ . \quad (25)$$

There are 10 kinematic FD of that type yielding the contribution

$$\begin{aligned}
&C_F\sigma_{21}L_{k_2}^2+\left(C_F-\frac{C_V}{2}\right)\sigma_{21}\left(L_{k_1}^2-L_{k_2}^2\right)\\
&+\frac{C_V}{2}\sigma_{21}\left(L_{k_1}^2-L_{k_2}L_{k_1}\right)+\sigma_a\sigma_{21}\sigma_a\left(L^2-Lk_1{}^2\right)\\
&+\frac{C_V}{2}\sigma_{21}\left(\ln^2\frac{s\alpha_1}{\lambda^2}-\ln^2\frac{s\alpha_1}{\boldsymbol{k}_1^2}\right)\\
&+\frac{C_V}{2}\sigma_{21}\left(\ln^2\frac{s\beta_2}{\lambda^2}-\ln^2\frac{s\beta_2}{\boldsymbol{k}_1^2}\right)\\
&+f_{k2b}f_{k1a}\sigma_b\sigma_a\left(\ln^2\frac{2k_1k_2}{\lambda^2}-\ln^2\frac{2k_1k_2}{\boldsymbol{k}_1^2}\right)\\
&-if_{ba1}\sigma_b\sigma_2\sigma_a\left(\ln^2\frac{s\beta_1}{\lambda^2}-\ln^2\frac{s\beta_1}{\boldsymbol{k}_1^2}\right)\\
&-if_{ba2}\sigma_b\sigma_1\sigma_a\left(\ln^2\frac{s\alpha_2}{\lambda^2}-\ln^2\frac{s\alpha_2}{\boldsymbol{k}_1^2}\right)\ . \quad (26)
\end{aligned}$$

Rearranging the color indices using the relations $[\sigma_a,\sigma_b]=if_{abc}\sigma_c$ and $f_{abk}f_{abm}=C_V\delta_{km}$, one can see that the factor accompanying the new color (exotic) structure $f_{a2b}f_{a1k}\sigma_b\sigma_k$ is exactly equal to zero (Fig. (3)). The cancellation of such a kind of exotics takes place in the higher orders of PT as well.

Hence, the result for the amplitude with two emitted gluons reads

$$\mathcal{M}^{(2)}=F_0^{(2)}B_{\rm QCD}^{(2)}\ , \quad (27)$$

where

$$\begin{aligned}
F_0^{(2)}&=\frac{p_1\cdot\epsilon_1}{p_1\cdot k_1}\frac{p_1\cdot\epsilon_2}{p_1\cdot k_2}g^2\ ,\\
B_{\rm QCD}^{(2)}&=\frac{\alpha_s}{2\pi}\left[C_F\ L^2+\frac{C_V}{2}\left(\ln^2\frac{\boldsymbol{k}_1^2}{\lambda^2}+\ln^2\frac{\boldsymbol{k}_2^2}{\lambda^2}\right)\right]\ . \quad (28)
\end{aligned}$$

Arguments in favour of exponentiation of higher orders of PT allow one to conclude that

$$\mathcal{M}_\infty^{(2)}=F_0^{(2)}\exp\left[-B_{\rm QCD}^{(2)}\right]\ . \quad (29)$$

Let us give some reasons for the following form of inelastic quark form factor with emission of m real hard gluons:

$$\mathcal{M}_\infty^{(m)}=\mathcal{M}_0^{(m)}\exp\left[-\frac{\alpha_s}{4\pi}\left(C_F\ L^2+\frac{C_V}{2}\sum_{i=1}^m\ln^2\frac{\boldsymbol{k}_i^2}{\lambda^2}\right)\right]\ , \quad (30)$$

where the amplitudes $B_0^{(m)}$ in the Born approximation are given by

$$B_0^{(m)} = g^m \prod_{i=1}^{m} \epsilon_\mu(k_i) j_\mu(k_i) \sum_{perm} \bar{u}(p_2)\Gamma \sigma_{a_1}...\sigma_{a_m} u(p_1) \ , \tag{31}$$

and the following ordering takes place

$$\alpha_{a_1} \gg \alpha_{a_2} \gg \cdots \gg \alpha_{a_n} \ , \quad \beta_{a_1} \ll \beta_{a_2} \ll \cdots \ll \beta_{a_n} \ , \quad s\alpha_{a_i}\beta_{a_i} = \boldsymbol{k_i}^2 \ . \tag{32}$$

The kinematic conditions (32) provide the extraction of the leading DL contributions to the integrated hard gluon distribution.

Consider now the ladder amplitude $\mathcal{M}_n^{(m)}$ with $m+1$ rungs, each of which with n_j virtual gluons ($\sum_{j=0}^m n_j = n$). According to magnitudes of their transversal momenta, these n virtual gluons can be separated in the following kinematic classes:

$$\boldsymbol{q}^2 \gg \boldsymbol{k}^2_{i_m} \gg \boldsymbol{k}^2_m, \quad \boldsymbol{k}^2_m \gg \boldsymbol{k}^2_{i_{m-1}} \gg \boldsymbol{k}^2_{m-1} \ , \\ \ldots, \quad \boldsymbol{k}^2_1 \gg \boldsymbol{k}^2_{i_0} \gg \lambda^2 \ , \tag{33}$$

with the ordering in each class

$$\boldsymbol{k}^2_{1_l} \gg \boldsymbol{k}^2_{2_l} \gg \cdots \gg \boldsymbol{k}^2_{n_{j_l}} \ , \quad l = 0, 1, \ldots, m \ .$$

As a result, we obtain

$$\mathcal{M}_n^{(m)} = \sum_{n_j} \prod_{i=1}^{m} \frac{1}{(n_i)!} \mathcal{M}_{(i)}^{n_i} = \frac{1}{n!} \left(\sum_{n=0}^{m} \mathcal{M}_{(n)} \right)^n \ , \tag{34}$$

and the statement (30) immediately follows from Eq. (34).

V. CONCLUSIONS AND OUTLOOK

We emphasize that the quark inelastic form factor with emission of m real hard gluons cannot be expressed in terms of the elastic form factor, in contrast to the QED case. Therefore, the Poisson distribution is violated in QCD. Nevertheless, it can be shown that the quantity

$$\sum_{m=0}^{\infty} \frac{1}{m!} \left| \mathcal{M}_\infty^{(m)} \right|^2 \prod_{i=1}^{m} \left(g^2 \frac{\mathrm{d}^3 k_i}{2\omega_i (2\pi)^3} \right) \tag{35}$$

does not depend on the external virtuality s that is, in fact, the consequence of the KLN theorem [16]. This statement can be considered as a generator of relations between certain contributions in each order of perturbative expansion. This sort of relations in one- and two-loop orders was studied in [12]. It is necessary to keep in mind that the decay type situation $\alpha_i \beta_j \approx \alpha_j \beta_i$ plays an important role in this check problem. In the present work, we have considered only the kinematics in which the emitted real gluons are ordered according to their transversal momenta

$$\boldsymbol{k}_1^2 \gg \boldsymbol{k}_2^2 \cdots \gg \boldsymbol{k}_n^2 \ , \tag{36}$$

which produces the DL contribution to the total cross section, but the decay kinematics drops out in this regime. The condition (36) can be formulated in a Lorentz invariant form using the relation

$$\boldsymbol{k}_i^2 = s\alpha_i\beta_i = 2\frac{k_i \cdot p_1 \ k_i \cdot p_2}{p_1 \cdot p_2} \ . \tag{37}$$

We believe that the result for inelastic form factor, Eq. (30), under the condition (36) can in principle be verified by relevant exclusive experiments.

It is worth noting that the arguments given above do not take into account the nature of external particle which can be any probing particle including scalar, pseudoscalar, vector and pseudovector particle ($\Gamma = 1; \gamma_5; \gamma_\rho; \gamma_5\gamma_\rho$). In particular, all the results are valid for the quark Pauli form factor. Moreover, just a small modification must be made when the flavor of one of the quarks changes. We will not consider this case here.

Let us emphasize that Gribov's idea on the pole-dominated contribution of virtual exchange particle with minimal transversal momentum can be effectively applied to any gauge theory including, *e.g.*, gravitation. In the latter case, the virtual exchange particles as well as the real emitted ones are gravitons.

We point out two possible applications of the results obtained in the present work. One concerns the decay of a heavy particle current described by Γ to the quark-antiquark pair accompanied by an arbitrary number of gluons. Here the form factor reveals itself in the time-like region. Another possible application is the mechanism of the jet formation in DIS experiments where the spacelike region can be probed.

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Appendix

In QCD, the 1-loop order of PT contribution to the elastic form factor in the scattering channel has the form

$$\mathcal{M}^{(1-loop)} = -\frac{ig^2}{(2\pi)^4}\int \frac{\mathrm{d}^4k}{(k^2-\lambda^2+i0)} \times \frac{\bar{u}(p_2)\gamma_\mu\sigma_a(\hat{p}_2-\hat{k}+m)\Gamma(\hat{p}_1-\hat{k}+m)\gamma_\mu\sigma_a u(p_1)}{((p_2-k)^2-m^2+i0)\,((p_1-k)^2-m^2+i0)} \ . \tag{38}$$

Simplifying the numerator and neglecting the power suppressed part (related to the Pauli form factor) one finds $2sC_FV_0$ in the numerator. Using the Sudakov parameterization of the loop momentum

$$k^2-\lambda^2+i0 = s\alpha\beta-\boldsymbol{k}^2-\lambda^2+i0, \tag{39}$$
$$(p_1-k)^2-m^2+i0 = -s\alpha(1-\beta)-\boldsymbol{k}^2+i0,$$
$$(p_2-k)^2-m^2+i0 = -s\beta(1-\alpha)-\boldsymbol{k}^2+i0 \ ,$$

and analyzing the location of poles in the α, β planes, one finds that the nonzero DL contribution arises from two situations corresponding to the location of the poles of the integrand in different half-planes of α-plane:

$$0<(\beta,\alpha)<1 \ , \quad s\alpha\beta>\lambda^2 \ , \tag{40}$$
$$0<(-\beta,-\alpha)<1 \ , \quad s\alpha\beta>\lambda^2 \ . \tag{41}$$

Performing the $\mathrm{d}^2\boldsymbol{k}$ integration

$$\int \frac{\mathrm{d}^2\boldsymbol{k}}{s\alpha\beta-\boldsymbol{k}^2-\lambda^2+i0} = -i\pi^2\int \mathrm{d}\boldsymbol{k}^2\delta(s\alpha\beta-\boldsymbol{k}^2-\lambda^2) = -i\pi^2\theta(s\alpha\beta-\lambda^2) \ , \tag{42}$$

we arrive at

$$2\int\limits_{\frac{\lambda^2}{s}}^{1}\frac{\mathrm{d}\alpha}{\alpha}\int\limits_{\frac{\lambda^2}{s}}^{1}\frac{\mathrm{d}\beta}{\beta}\theta(s\alpha\beta-\lambda^2) = \ln^2\frac{s}{\lambda^2} \ , \tag{43}$$

which immediately yields the one-loop amplitude in the double-log approximation:

$$\mathcal{M}^{(1-loop)} = -V_0\ C_F\frac{\alpha_s}{4\pi}\ln^2\frac{s}{\lambda^2} \ . \tag{44}$$

The QED result can be obtained from Eq. (44) by the replacement $C_F\to 1$.

In order to calculate the form factor (38) in the case when the transversal momentum of the emitted real gluon is taken as an upper bound for the virtual boson transversal momentum, one needs to take into account another θ-function: $\theta\left(s\alpha\beta-k_\perp^2\right)$ in the loop integration. Then, one gets

$$2\int\limits_{\frac{\lambda^2}{s}}^{1}\frac{\mathrm{d}\alpha}{\alpha}\int\limits_{\frac{\lambda^2}{s}}^{1}\frac{\mathrm{d}\beta}{\beta}\left[\theta(s\alpha\beta-\lambda^2)-\theta\left(s\alpha\beta-k_\perp^2\right)\right] = \ln^2\frac{s}{\lambda^2}-\ln^2\frac{s}{k_\perp^2} \ , \tag{45}$$

thus obtaining the contribution of the "down" loops in Eq. (12).

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